Countable and Pointwise Countable Type Families

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Abstract

The present work continues the research on families of subspaces of a given topological space possessing a particular topological property but in a collective manner and started as a joint study with M. Choban. Families of A($\mathcal{P}$) - subspaces, of p-subspaces, and strict p-subspaces were regarded in [4]. Families with the first axiom of countability were defined and studied in [5]. All families mentioned above are generalizations of the k-metrizable families, presented and studied in [2].

More precisely families of countable and poinwise countable types are considered in this report. It is shown that these families are open continuous images of k-metrizable families. Moreover a countable type family $\mathcal{A}$ is such an image under a mapping that is an $\mathcal{A}$-covering. Namely:

Theorem 0.1. *Let $\mathcal{A}$ be a family of subspaces of a regular space X, and $X = \cup\mathcal{A}$. The following assertion are equivalent:*

i. $\mathcal{A}$ is a family of subspaces of countable type;

ii. There exist a regular space Z and an open continuous $\mathcal{A}$-covering mapping $f : Z \longrightarrow X$ of the space Z onto the space X such that $\mathcal{A}^{-1} = \{f^{-1}(L) : L \in \mathcal{A}\}$ is a family of subspaces of Z of countable type and f is an $\mathcal{A}$-covering.

iii. There exist a regular space Z, a continuous pseudometric d on Z and an open continuous mapping $f : Z \longrightarrow X$ of the space Z onto the space X such that $\mathcal{A}^{-1} = \{f^{-1}(L) : L \in \mathcal{A}\}$ is k-metrizable by the pseudometric d and f is an $\mathcal{A}$-covering.

Note that the notion of a family of countable type is a generalization of a countable type space defined and studied by A. Arhangel'skii [1].

A selection theorem is obtained, too. In detail, for a lower semi-continuous mapping θ from a paracompact F_σ-discrete space Y into a regular space X there exists a multi-valued upper semi-continuous selection for θ, if the images of the points of Y are elements of a pointwise countable type family $\mathcal{A}$ of X.

In the proof of the above mentioned selection theorem a reduction principle of M. Choban [3] is applied. The Selection theory of set-valued mappings began with the famous theorem of E. Michael in 1956. Usually in such results the range space is metrizable and the domain is paracompact. The main objective is to extend such selection results under more general requirements for the range space.

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